Quaternions of Hamilton and Tait, the *Ausdehnungslehre* of Grassmann, the vector analysis of Gibbs and Heaviside. It is this problem of how to harmonise, unify, generalise, and extend that I have been studying. Analysts and physicists dislike Mr. McAulay's idea of an independent plant; they prefer to cultivate the old tree venerable with the growth of ages.

After studying impartially all the writers at my command I came to the conclusion that the analysis of vectors is complementary to the analysis of versors, and that the fundamentel rules for the former are:—

$$i^2 = + \qquad j^2 = + \qquad k^2 = + \qquad (1)$$
$$ij = -ji \qquad jk = -kj \qquad ik = -ki \qquad (2)$$
$$ij = k \qquad jk = i \qquad ki = j \qquad (3);$$

whereas for the latter they are:—

$$i^{\overset{\pi}{2}} i^{\overset{\pi}{2}} = - \qquad j^{\overset{\pi}{2}} j^{\overset{\pi}{2}} = - \qquad k^{\overset{\pi}{2}} k^{\overset{\pi}{2}} = - \qquad (4)$$
$$i^{\overset{\pi}{2}} j^{\overset{\pi}{2}} = -j^{\overset{\pi}{2}} i^{\overset{\pi}{2}} \quad k^{\overset{\pi}{2}} j^{\overset{\pi}{2}} = -j^{\overset{\pi}{2}} k^{\overset{\pi}{2}} \quad i^{\overset{\pi}{2}} k^{\overset{\pi}{2}} = -k^{\overset{\pi}{2}} i^{\overset{\pi}{2}} \quad (5)$$
$$i^{\overset{\pi}{2}} j^{\overset{\pi}{2}} = -k^{\overset{\pi}{2}} \qquad j^{\overset{\pi}{2}} k^{\overset{\pi}{2}} = -i^{\overset{\pi}{2}} \qquad k^{\overset{\pi}{2}} i^{\overset{\pi}{2}} = -j^{\overset{\pi}{2}} \qquad (6)$$

It follows that in the manipulation of the products of vectors, the distributive rule applies but not the associative; while in the products of versors both apply. These fundamental rules for vectors are based on physical considerations, the principal one of which is that the square of a vector is essentially positive, whereas, according to quaternionists, it is essentially negative. My view agrees with that principle of analysis which considers the cosine in the first and fourth quadrants to be positive; to make it negative produces confusion and error. These principles harmonise with those of Gibbs and Heaviside; and in the memoir quoted I have carried them out to their logical development. It is this development which Prof. Knott characterises as "a pseudo-quaternionic system of vector algebra, which is non-associative in its products." I see no worthy aim in being canny about the matter; my sole aim was to develop the system so that its truth or falsity might the more readily appear. At the end of his article Prof. Knott admits that the *assumption* that the square of a unit vector is positive unity leads to an algebra which is essentially different from the algebra of quaternions. As regards the fundamental principle being an assumption, I refer him to that same chapter of "Kelland and Tait" which he quotes, where he will find, italics and all:—"We retain what Sir Wm. Hamilton terms the *associative laws of multiplication*: the law which assumes that it is indifferent in what way operations are grouped, provided the order be not changed; the law which makes it indifferent whether we consider $a\,b\,c$ to be $a \times b\,c$ or $a\,b \times c$. This law is *assumed* to be applicable to multiplication in its new aspect (for example that $i\,j\,k = i\,j.k$) and being assumed it limits the science to certain boundaries, and, along with other assumed laws, furnishes the key to the interpretation of results. The law is by no means a necessary law. Some new forms of the science may possibly modify it hereafter. In the meantime the assumption of the law fixes the limits of the science." Here an authoritative expounder places the quaternion algebra on precisely the same footing that Dr. Knott places the "pseudo-quaternionic;" and he even predicts that in the course of time such a complementary algebra will be developed. It is incumbent on a critic, having admitted the logical development, to show that the assumptions are absurd, or correspond to nothing in physical science; instead of which he informs us that he is appalled by the complexity, but nevertheless he feels sure that it contains nothing new. As regards newness I invite his attention to pr. 93 of the "Principles," where I have investigated the rules for the several partial products of any number of vectors in space of not more than four dimensions (and they may be easily extended to space of higher dimensions). These consist of certain rules of reduction which are to be taken along with the rule of signs of determinants, thus embracing determinants and Grassmann's combinatory products in the general theory of products of vectors. He will also find there some reasons for believing that the triad of rules No. 3 are very different in nature from the other two triads, Nos. 1 and 2. It is possible to get along without No. 3.

That vectors should be treated vectorially, and versors versorially, and rotors rotorially, is neither nonsense nor a truism. It is an important maxim, and of growing importance in these days. Violation of it has produced the fundamental weakness of Hamilton's analysis. In a more recent paper I have published the generalisations for space of the exponential, binomial, multinomial, and other fundamental theorems of analysis, and I show that it was from treating versors vectorially that Hamilton failed to discover them.

Prof. Knott defines a quaternion as the quotient of two vectors. Why choose the quotient; is not the product always the simpler idea? But further on vectors are identified with quadrantal quaternions, from which it follows that a quaternion is the quotient of two quadrantal quaternions. I have devoted some attention to logic; but I fail to extract any meaning out of this implicit definition.

Prof. Knott informs the reader that whereas Heaviside and myself find that $\nabla^2 u = d^2u/dx^2 + d^2u/dy^2 + d^2u/dz^2$ the real $\nabla^2 u$ is minus that quantity; but he does not explain why Prof. Tait prefers the unreal $\nabla^2 u$ in his "Treatise on Natural Philosophy." A scientific critic would, instead of using exclamation points, proceed to show that in every case $\nabla(\nabla\omega) = (\nabla\nabla)\,\omega$. If that can be proved, not from any fancied properties of italic letters, but from physical considerations, then I shall readily admit that ∇ behaves as a versor rather than a vector. The *onus probandi* lies on the minus men.

Austin, Texas, May 6. ALEXANDER MACFARLANE.

An Atmospheric Phenomenon in the North China Sea.

DURING a recent wintry cruise in H.M.S. *Caroline* in the North China Sea, a curious phenomenon was seen which may be of interest to your readers. The ship was on passage between Shanghai and the western entrance of the famous inland sea of Japan. On 24th February, at 10 p.m., when in latitude 32° 58′ N., longitude 126° 33′ E., which, on reference to the map, will be seen to be sixteen to seventeen miles south of Quelpart island (south of the Korean peninsula) some unusual lights were reported by the officer of the watch between the ship and Mount Auckland, a mountain 6,000 feet high. It was a windy, cold, moonlight night. My first impression was that they were either some fires on shore, apparently higher from the horizon than a ship's masthead, or some junk's "flare up" lights raised by mirage. To the naked eye they appeared sometimes as a mass; at others, spread out in an irregular line, and, being globular in form, they resembled Chinese lanterns festooned between the masts of a lofty vessel. They bore north (magnetic), and remained on that bearing until lost sight of about midnight. As the ship was passing the land to the eastward at the rate of seven knots an hour, it soon became obvious that the lights were not on the land, though observed with the mountain behind them.

On the following night, February 25th, about the same time, 10 p.m., the ship having cleared Port Hamilton, was steering east, on the parallel of 34°, when these curious lights were again observed on the same bearing, at an altitude of 3° or 4° above the horizon. It was a clear, still, moonlight night, and cold. On this occasion there was no land in sight on a north bearing when the lights were first observed, but soon afterwards a small islet was passed, which for the time eclipsed the lights. As the ship steamed on at a rate of seven knots an hour, the lights maintained a constant bearing (magnetic) of N. 2° W., as if carried by some vessel travelling in the same direction and at the same speed. The globes of fire altered in their formation as on the previous night, now in a massed group, with an outlying light away to the right, then the isolated one would disappear, and the others would take the form of a crescent or diamond, or hang festoon-fashion in a curved line. A clear reflection or glare could be seen on the horizon beneath the lights. Through a telescope the globes appeared to be of a reddish colour, and to emit a thin smoke.

I watched them for several hours, and could distinguish no perceptible alteration in their bearing or altitude, the changes occurring only in their relative formation, but each light maintained its oval, globular form.

They remained in sight from 10 p.m. until daylight (about 5.30 a.m.). When lost sight of the bearing was one or two points to the westward of north. At daylight land 1300 feet high was seen to the north and north-north-west, distant fifty miles, the mirage being extraordinary.

Thus, these lights were seen first in longitude 126° 33′ E., and last in longitude 128° 29′ E. At first the land was behind them, but during the greater part of the distance run it was forty-five or fifty miles away to the north; and the bearing of the lights for at least three-fourths of the distance did not change.

On arrival at Kobé I read in a daily paper that the "Unknown

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light of Japan" had, as was customary at this season of the year when the weather is very cold, stormy, and clear, been observed by fishermen in the Shimbara Gulf and Japanese waters. The article went on to say that these lights were referred to in native school-books, and attributed to electrical phenomena. On mentioning the matter, however, to the leading Europeans in Yokohama and Tokio, they appeared to have no knowledge of the matter.

Captain Castle, of H.M.S. *Leander*, informed me that, not long ago, the officers of his ship saw lights in the same locality which they thought at first were caused by a ship on fire. The course of the vessel was altered at once with a view of rendering assistance, but finding that the lights increased their altitude as he approached, he attributed them to some volcanic disturbance, and being pressed for time, resumed his course.

The background of high land seen on the first night dispels all idea of these extraordinary lights being due to a distant volcano. The uniformity of the bearing renders the theory of their being fires on the shore most improbable. I am inclined to the belief that they were something in the nature of St. Elmo's fires. It is probable that there are travellers among the readers of your interesting journal who have seen or heard of this phenomenon, and will be able to describe its origin and the atmospheric conditions necessary for its appearance.

CHAS. J. NORCOCK.

H.M.S. *Caroline*, Hongkong, April 10.

The Greatest Rainfall in Twenty-four Hours.

IN NATURE, May 4, Mr. Clement Wragge, of Brisbane, confidently asserts that Queensland has beaten the world's record in the extraordinary amount recorded on February 3, viz., 35·7 inches. I am sorry to have to take away such an unenviable palm from Queensland, by recalling a fact well known to every Indian meteorologist that the highest record extant belongs to Chirapunji, in the Khasia hills, where on June 14, 1876, 40·8 inches were recorded in the twenty-four hours. Not only so, but on the 12th 30 inches fell, and in the four days, from the 12th to the 15th inclusive, as much as 102 inches. Of course the effects were not so disastrous in this case, as indeed such a state of things is little removed from the normal at Chira in the early part of June, but I have a very clear recollection of it as I was at Chirapunji on the 12th and 13th, and not far from it on the memorable 14th.

The conditions which have occurred in Queensland and the North Island of New Zealand during the last six months have been a remarkable example of persistent abnormals, and though the total number of rational causes may still be wanting to explain everything, one or two were evidently in operation when I was there from October to January, and I am confident that from the empirical law of persistency, coupled with a few rational inferences, a forecast of impending floods could have been made and can be made for the future, much in the same way as the general character of the monsoon can be foretold in India.

May 13. E. DOUGLAS ARCHIBALD.

A Dust-whirl or (?) Tornado.

IN NATURE (vol. xl. p. 174) you kindly allowed me to describe a dust-whirl seen to originate on a heated dust-covered highway. The phenomenon has just been repeated under much similar circumstances, only in this instance the column of dust after oscillating to and fro on the highway for about half a minute, moved rapidly away in a curvilinear path in a northerly direction, the lower end of the whirl catching up loose material in its track where it touched the ground, which it did at intervals of from ten to fifteen yards, carrying the strawy litter from a strawberry bed upwards of 50 yards in the air. It appeared to dissipate into the upper air when crossing a meadow some 300 yards from its place of origin. The characteristic "swish" of the rushing air was very marked, and the four motions common to all tornadoes (see Lieut. Finley's "Character of Six Hundred Tornadoes"), viz. whirling from right to left, progressive motion to the north, a curvilinear track, and the dipping up and down, were all distinctly traced. The question therefore, naturally arises—Can these dust-whirls be tornadoes in miniature?

Conditions at the time of the occurrence:—Date, Thursday, May 11, 1893; time, 11 a.m. Corrected barometer, 30.327 (falling slightly). Dry bulb, 66°.5; wet, 51°.8 = rel. hum. 38 per cent. Wind, south; force, 1. Some upper cirrus radiating from north-east, and drifting slowly from north-west, showing top and bottom arcs of halo at 10 a.m. Black bulb *in vacuo* 128°.2; weather very warm and dry.

Driffield, May 11. J. LOVEL.

What becomes of the Aphis in the Winter?

I HAVE spent many weeks this spring closely observing the budding trees, with the object of discovering in what condition of life the aphis spends the winter; as the result of my observations, which were made under the microscope, I believe that the aphidæ during the autumn (or as many of them as have reached the state of reproduction) attach themselves to the stem of the tree, with their young inside them, in much the same way as the female members of the closely-allied family coccidæ do. In course of time the mother-aphis becomes simply a dried skin serving as a protection to the young. When the warm days of spring come these are developed and easily make their way through the skin and crawl on to the young leaves, there to begin their work of sucking and reproduction.

T. A. SHARPE.

Soot-figures on Ceilings.

MAY I suggest a distinct, if not an alternative cause for Prof. E. B. Poulton's soot figures in NATURE, April 27th? The ceiling plaster is very porous, except where it is in contact with the joists, etc. At such points very little deposit occurs compared with the spaces where the hot air is vigorously diffusing through into the cold space above. I suggest this because I am very familiar with a large ceiling where the rafters are thus picked out in light shades. Even the laths are picked out, but less distinctly. The main bolts likewise show dark, as in Prof. Poulton's sketch, as if there were an air-space by them. There is no perceptible difference in the figures near the central chandelier from those in the corners remotest from heating causes. The bombarding pattern is often very well shown where super-heated water pipes run along a white-washed wall. The effect of every little break, even a nail in the wall, is most striking. J. EDMUND CLARK.

A Difficulty in Weismannism Resolved.

IN my letter of the 1st inst. an omission of parentheses and quotation marks, which I omitted to note on the proof, alters the sense of the paragraph with quotations from the "Germ Plasm," pp. 434–5. It should be as follows:—"The note runs thus: 'Compare Marcus Hartog, NATURE, vol. xliv. p. 102,' (the reference omits my letter of Oct. 31, 1891). 'The deductions made by this author are logically correct but are no longer justifiable, since I myself have gained further insight into the problems concerned.'" The absence of the inverted commas disguises this recognition by Weismann of the validity of my objections, and of the consequent change in his own views.

Cork, May 15. MARCUS HARTOG.

NOTES.

THE Hon. Ralph Abercromby has given to the Royal Society of New South Wales the sum of £100, which is to be offered as prizes with the object of bringing about exhaustive studies of certain features of Australian weather. So far only one feature has been selected, and a prize of £25 is now offered for an exhaustive study of the well-known "Southerly Burster." It is understood that no essay which does not deal fully with the following points will be considered:—(1) The motions of the various strata of clouds for some hours preceding, at the time of, and following the "burster;" (2) the weather conditions which lead up to and follow the "burster," with weather charts of Australia for the day of occurrence and the following day; (3) the general conditions which modify the character of the "burster;" (4) The area of the "burster" and its track; (5) barograph traces showing the changes of pressure during the "burster;" (6) the direction and character of wind preceding it; (7) the relation of "bursters" to rainfall. The essay must not exceed 50 pages of foolscap, and must be sent in not later than March 31, 1894. It must embody studies of several "bursters," and must be chiefly the result of original research of the author, but authors

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